Journal of Sound and Vibration (1998) **217**(5), 945–949
Article No. sv981787

LETTERS TO THE EDITOR

FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF SIMPLY SUPPORTED AND CLAMPED PLATES OF REGULAR POLYGONAL SHAPE WITH A CONCENTRIC CIRCULAR SUPPORT

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(*Received* 22 *June* 1998)

1. Introduction

The present study deals with the determination of the fundamental frequency of transverse vibration of the plate system depicted in Figure 1 for two types of boundary conditions at the outer edge: simply supported and clamped. Apparently no solutions are available in the open literature [1]. The case of circular, solid and annular plates with a concentric circular support has been treated rather recently [2, 3].

This study deals with the solution of the title problem using the optimized Rayleigh–Ritz method [4] coupled with the conformal mapping approach which allows for the use of simple co-ordinate functions which, in the transformed plane, satisfy the essential boundary conditions of the mechanical system [5]. In the case of a square plate an independent solution is obtained by means of the finite element method using a very modern, efficient code [6].

2. Approximate solution by means of conformal mapping—Rayleigh–Ritz approach

The solution of the plate vibration problem is governed by the classical functional

$$J(W) = D \iint_P [(W_{xx} + W_{yy})^2 - 2(1-\nu)(W_{xx}W_{yy} - W_{xy}^2)]\,\mathrm{d}x\,\mathrm{d}y$$

$$- \rho h \omega^2 \iint_P W^2\,\mathrm{d}x\,\mathrm{d}y, \qquad (1)$$

subject to appropriate boundary conditions. As is well known the regular polygonal shape of degree s is transformed onto a unit circle in the ζ-plane by means of the Schwarz–Christoffel transformation (Figure 1)

$$z = f(\zeta) = A_s a_p F(\zeta) = A_s a_p \int_0^C \frac{\mathrm{d}\zeta}{(1+\zeta^s)^{2s}}, \qquad (2)$$

where $z = x + yi$ and $\zeta = \xi + \eta i$.

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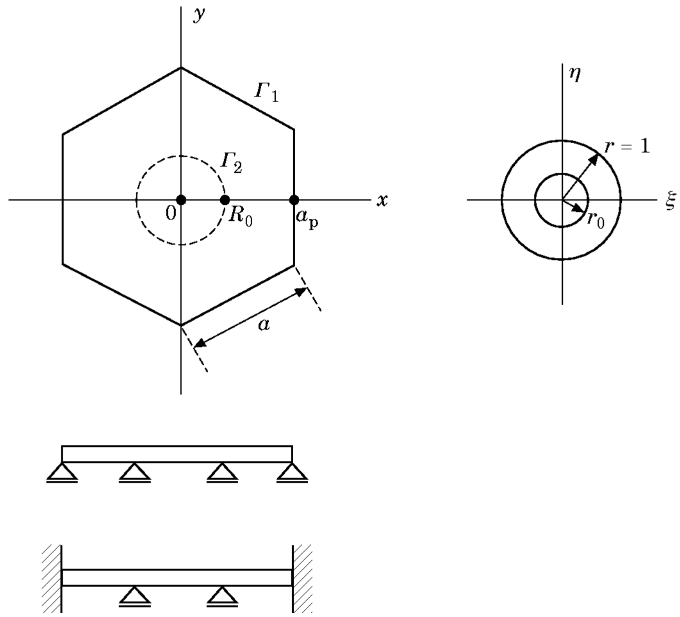

Figure 1. Vibrating mechanical system under study.

The parameter A_s is a function of the degree of the polygon, s, and has been tabulated in the open literature [5]. The displacement amplitude $W(x, y)$ will be approximated in the ζ-plane by means of a functional relation expressed as a summation of polynomials and where the azimuthal variation is disregarded [5].

In the present study W_a has been expressed as

$$W_a = C_1(\alpha_1 r^p + \beta_1 r^2 + 1) + C_2(\alpha_2 r^{p+1} + \beta_2 r^2 + 1) + C_3(\alpha_3 r^{p+2} + \beta_3 r^2 + 1) \quad (3)$$

in the case of simply supported plates, where the α_i's and β_i's are determined satisfying the essential conditions (Figure 1)

$$W_a(1) = W_a(r_0) = 0 \quad (4)$$

and p is the optimization parameter [4].

On the other hand, in the case of a clamped, outer boundary the displacement amplitude is approximated using

$$W_a = C_1(\alpha_1 r^p + \beta_1 r^3 + \gamma_1 r^2 + 1) + C_2(\alpha_2 r^{p+1} + \beta_2 r^3 + \gamma_2 r^2 + 1) + C_3(\alpha_3 r^{p+2} + \beta_3 r^3 + \gamma_3 r^2 + 1), \quad (5)$$

where the α_i's, β_i's and γ_i's are evaluated by substituting each co-ordinate function in the governing essential boundary conditions (Figure 1)

$$W_a(1) = W_a'(1) = W(r_0) = 0. \quad (6)$$

The value of r_0 is approximately determined, when $R_0/a_p \ll 1$ by means of the expression [5]

$$r_0 = R_0/a_p A_s. \quad (7)$$

In view of the fact that $W_a = W_a(r)$, the governing functional (1) results, in the ζ-plane, in

$$\frac{A_s^2 a_p^2}{D} J(W_a) = \iint_c \left\{ \frac{1+\nu}{2} \frac{(W_{ar^2} + W_{ar}/r)^2}{|F'(\zeta)|^2} \right.$$

$$\left. + \frac{1-\nu}{2} \frac{|(W_{ar^2} - W_{ar}/r)\, \mathrm{e}^{-2\theta i} F'(\zeta) - 2W_{ar}\, \mathrm{e}^{-\theta i} F''(\zeta)|^2}{|F'(\zeta)|^4} \right\} r\, \mathrm{d}r\, \mathrm{d}\theta$$

$$- \frac{A_s^4 \cot^4 \pi/s}{16} \Omega^2 \iint_c W_a^2 |F'(\zeta)|^2 r\, \mathrm{d}r\, \mathrm{d}\theta, \tag{8}$$

where

$$|F'(\zeta)|^2 = 1/|1 + \zeta^s|^{4/s} = 1/(1 + 2r^s \cos s\theta + r^{2s})^{2/s}, \tag{9}$$

$$\Omega^2 = (\rho h/D)\omega^2 a^4, \qquad a_p = (a/2) \cot \pi/s; \tag{10, 11}$$

a is the side of the polygon (Figure 1).

Substituting equations (3) or (5) in equation (8) and applying the classical Rayleigh–Ritz method one obtains a linear, homogeneous system of equations in the C_i's. The non-triviality condition yields a secular determinant whose lowest root constitutes the fundamental frequency coefficient under investigation, $\Omega_1 = \sqrt{\rho h/D}\omega_1 a^2$.

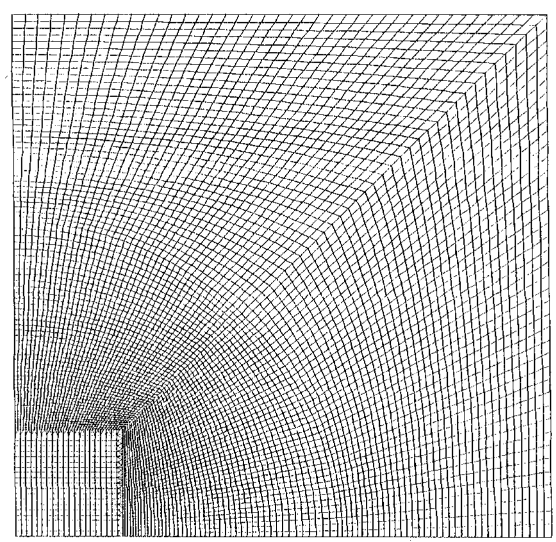

Figure 2. Finite element mesh for the configuration characterized by $R_0/a_p = 0{\cdot}6$ in the case of a square plate with a concentric circular support.

TABLE 1

Fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D}\omega_1 a^2$ *of the plate structural system shown in Figure* 1

Outer boundary conditions	R_0/a_p	Square: Analyt. solution	Square: Finite element solution	Pentagonal	Hexag.
Simply supported	0·1	57·94	57·64	32·22	20·76
	0·2	67·70	64·83	37·82	24·44
	0·3	81·10	77·00	45·63	29·65
	0·4	97·93	91·25	55·16	35·91
	0·5	103·38	98·90	56·54	36·26
	0·6	86·25	83·40	46·04	29·22
Clamped	0·1	86·85	85·10	47·95	31·09
	0·2	101·13	98·11	56·16	36·53
	0·3	121·36	116·33	67·75	44·21
	0·4	138·38	132·31	76·28	49·32
	0·5	117·35	116·72	62·77	39·93
	0·6	88·89	88·98	47·25	29·96

3. FINITE ELEMENT RESULTS

An independent solution was obtained for the case of the square plate with a concentric circular support, using ALGOR [6]. Figure 2 depicts the finite element mesh for one quarter of the structural system. The number of elements varied as a function of the complexity of the configuration. For $R_0/a_p = 0{\cdot}5$, for instance, 3584 elements were used and the number of nodes totalled 3641. For $R_0/a_p = 0{\cdot}6$ the configuration consisted of 6879 elements and 6961 nodes; see Figure 2.

4. NUMERICAL RESULTS

Table 1 depicts values of Ω_1 for square, pentagonal and hexagonal plates with a concentric circular support. As it was expected, the condition at the outer boundary does not carry significant weight as the inner support is placed closer to the outer boundary, e.g., for $R_0/a_p \geqslant 0{\cdot}6$. In the case of the square plate the finite element results (presumably very accurate) are in reasonably good engineering agreement with the analytical values of Ω_1, the maximum difference being of the order of 7% in the case of a simply supported plate, for $R_0/a_p = 0{\cdot}4$. One should recall, at this point, that the natural boundary condition at the outer boundary is not satisfied when using the co-ordinate functions defined in equation (3).

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program, by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur and by Comisión de Investigaciones Científicas, Buenos Aires Province.

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